

Time evolution of two distant SQUID rings irradiated with entangled electromagnetic field

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Introduction

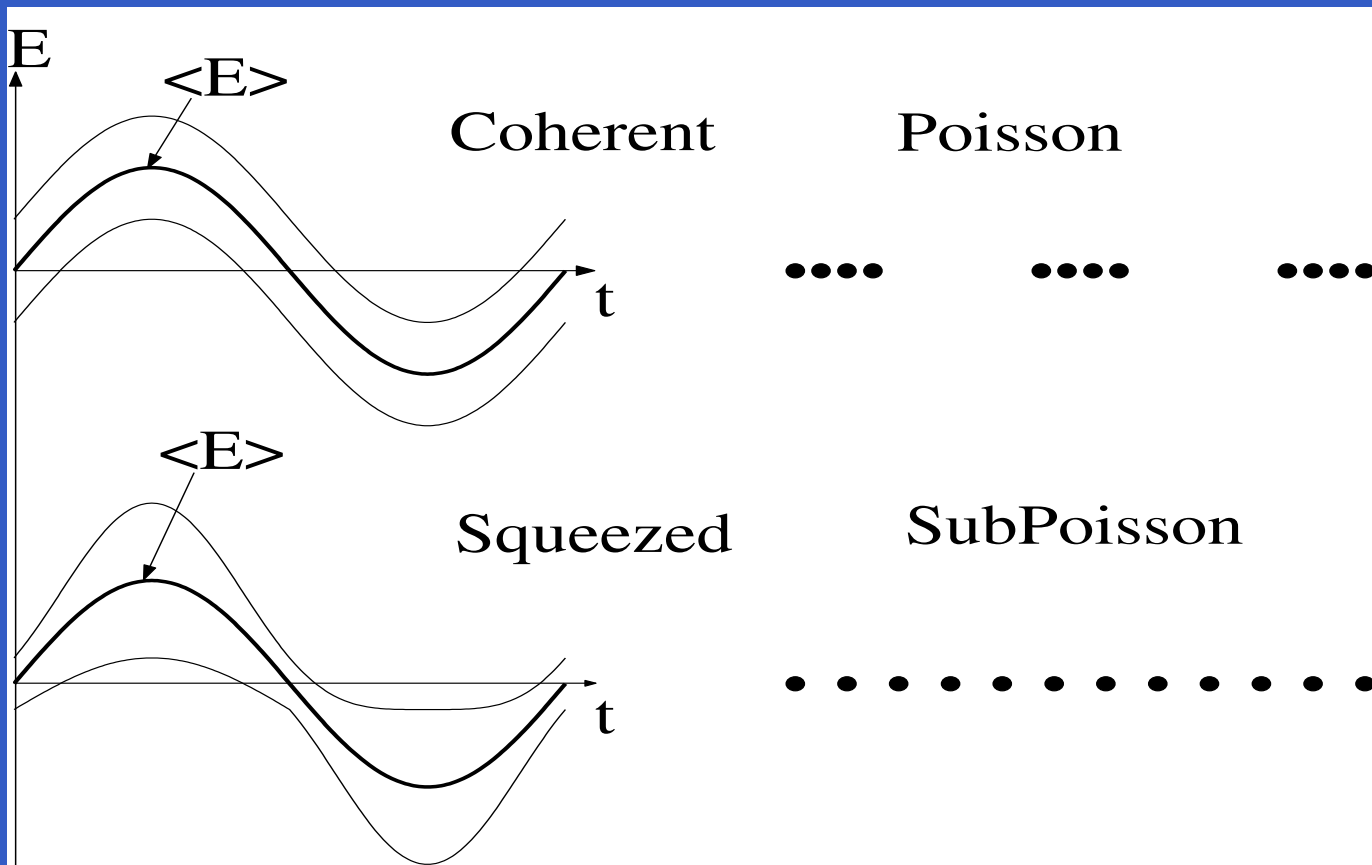


Figure 1: Coherent (top) and Squeezed (bottom)



Experiment

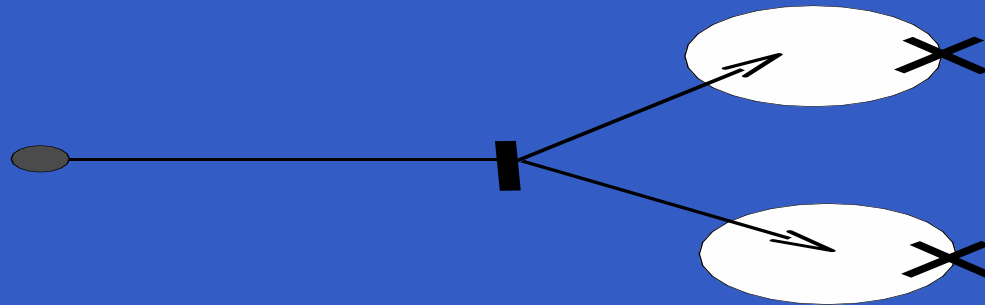


Figure 2: Correlated microwaves produced by the same source irradiate two distant SQUID rings.

Objective

The purpose is to demonstrate that although at

$$t = 0; \quad \rho_S(0) = \rho_A(0) \otimes \rho_B(0) \quad (1)$$

at a later time t , in general

$$\rho_S(t) \neq \rho_A(t) \otimes \rho_B(t) \quad (2)$$

and therefore the two SQUID rings are correlated although they are apart from each other.



Objective

In previous work electron interference experiments have been considered; that work was in the *external field approximation* where back reaction from the electrons on the em field is neglected.



Objective

Extension in current work is to take into account the back reaction; we consider the Hamiltonian of the whole system and study its temporal evolution.



Hamiltonian

$$H = \frac{\Delta_A}{2}\sigma_z^A + \frac{\Delta_B}{2}\sigma_z^B + \omega_1\left(\alpha_1\alpha_1^\dagger + \frac{1}{2}\right) + \omega_2\left(a_2a_2^\dagger + \frac{1}{2}\right) + \lambda_A\sigma_x^A(\alpha_1 + \alpha_1^\dagger) + \lambda_B\sigma_x^B(a_2 + a_2^\dagger). \quad (3)$$

$\Delta_{A(B)}$ is the energy gap of the first (second) SQUID

$\omega_{1(2)}$ is the frequency of the first (second) mode of microwaves



Hamiltonian

$$H = \frac{\Delta_A}{2}\sigma_z^A + \frac{\Delta_B}{2}\sigma_z^B + \omega_1\left(\alpha_1\alpha_1^\dagger + \frac{1}{2}\right) + \omega_2\left(a_2a_2^\dagger + \frac{1}{2}\right) + \lambda_A\sigma_x^A(\alpha_1 + \alpha_1^\dagger) + \lambda_B\sigma_x^B(a_2 + a_2^\dagger). \quad (4)$$

$\lambda_{A(B)}$ is the coupling between SQUID A(B) and the first (second) mode of microwaves. parameters in units where $\hbar = c = k_B = 1$.



System Evolution

Assuming that at $t = 0$ system is described by

$$\rho(0) = \rho_A \otimes \rho_B \otimes \rho_F \quad (5)$$

we have calculated numerically the time evolution of the system as

$$\rho(t) = e^{iHt} \rho(0) e^{-iHt} \quad (6)$$



System Evolution

Furthermore, with partial traces calculated the following reduced density matrices:

$$\rho_S(t) = \text{Tr}_{F_1, F_2} \rho(t)$$

$$\rho_A(t) = \text{Tr}_B \rho_S(t)$$

$$\rho_B(t) = \text{Tr}_A \rho_S(t)$$

$$\rho_F(t) = \text{Tr}_{A, B} \rho(t)$$

$$\rho_{F_1}(t) = \text{Tr}_{F_2} \rho_F(t)$$

$$\rho_{F_2}(t) = \text{Tr}_{F_1} \rho_F(t)$$

(7)



Calculated Quantities

In order to demonstrate this explicitly, we have calculated $\langle N_A N_B \rangle$ and $\langle N_A \rangle \langle N_B \rangle$, in terms of the 'average level'

$$\begin{aligned}\langle N_A N_B \rangle &= \text{Tr}[\rho_S(\sigma_z^A \otimes \sigma_z^B)] \\ \langle N_A \rangle &= \text{Tr}[\rho_A \sigma_z^A] \\ \langle N_B \rangle &= \text{Tr}[\rho_B \sigma_z^B]\end{aligned}\tag{8}$$



Calculated Quantities

We have also calculated

$$\begin{aligned}\langle I_A I_B \rangle &= \text{Tr}[\rho_S(\sigma_x^A \otimes \sigma_z^B)] \\ \langle I_A \rangle &= \text{Tr}[\rho_{S_1} \sigma_x^A] \\ \langle I_B \rangle &= \text{Tr}[\rho_{S_2} \sigma_x^B]\end{aligned}\quad (9)$$



Calculated Quantities

Furthermore, calculated the expectation value of the magnetic flux $\alpha_i + \alpha_i^\dagger$ ($i=1,2$) induced by the two field modes, as follows:

$$\begin{aligned}\langle \Phi_1 \Phi_2 \rangle &= Tr\{\rho_F[(\alpha_1 + \alpha_1^\dagger) \otimes (\alpha_2 + \alpha_2^\dagger)]\} \\ \langle \Phi_1 \rangle &= Tr[\rho_{F_1}(\alpha_1 + \alpha_1^\dagger)] \\ \langle \Phi_2 \rangle &= Tr[\rho_{F_2}(\alpha_2 + \alpha_2^\dagger)].\end{aligned}\tag{10}$$



We consider two initial states for the microwaves:

- the separable (mixed) state

$$\rho_{\text{sep}} = \frac{1}{2}(|01\rangle\langle 01| + |10\rangle\langle 10|) \quad (11)$$



We consider two initial states for the microwaves:

- the entangled state

$$\rho_{\text{ent}} = \rho_{\text{sep}} + \frac{1}{2}(|01\rangle\langle 10| + |10\rangle\langle 01|), \quad (12)$$



We consider two initial states for the microwaves:

- the entangled state

$$\rho_{\text{ent}} = \rho_{\text{sep}} + \frac{1}{2}(|01\rangle\langle 10| + |10\rangle\langle 01|), \quad (13)$$

Note that ρ_{ent} and ρ_{sep} differ only by the nondiagonal elements.



Note that:

- Two SQUID rings are approximated as two-level systems.
- The infinite dimensional Hilbert spaces of the two microwave modes are truncated at N_{max} which is taken to be much greater than the average value of photons in the microwaves.



Note that:

- As a measure of accuracy of the approximation we calculated, at a generic instant time t , the traces of the truncated matrices. In the limit $N_{max} \rightarrow \infty$ they equal to 1; and in the truncated case they should be very close to 1. In all our results the traces were greater than 0.98.



Parameters

Values of the parameters compatible with the current experimental setup have been chosen as

$$\omega_1 = \omega_2 = \Delta_A = \Delta_B = 1.3 * 10^{-4},$$
$$\lambda_A = \lambda_B = 4.2 * 10^{-5} \text{ in units where}$$
$$\hbar = c = k_B = 1$$

In all our examples we have chosen

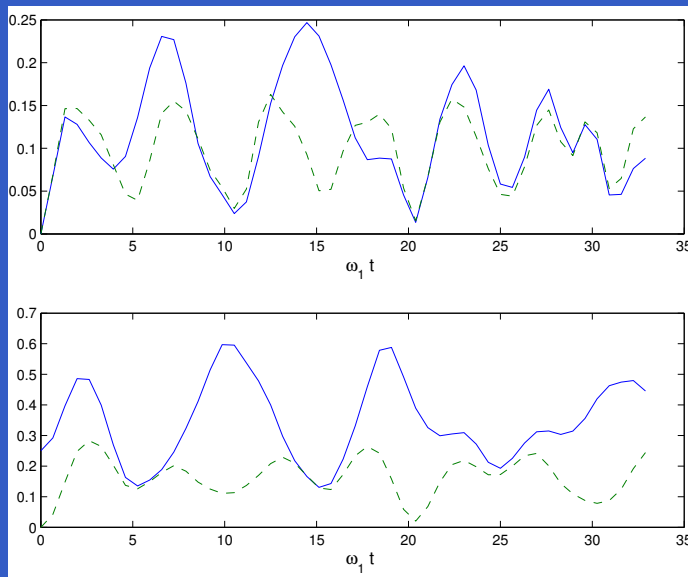
$$\rho_S(0) = |g\rangle\langle g| \otimes |e\rangle\langle e| \quad (14)$$

where $|g\rangle$ and $|e\rangle$ are the ground and excited states of the SQUID rings, respectively.



Model - Analysis

Top graph:

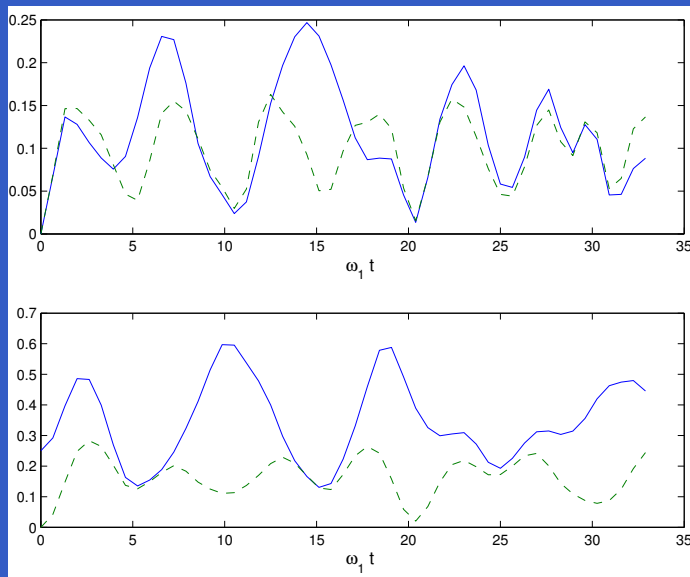


$\langle N_A N_B \rangle$ (solid line) and $\langle N_A \rangle \langle N_B \rangle$ (broken line) as functions of dimensionless time $\omega_1 t$. The results show that $\langle N_A N_B \rangle$ is different from $\langle N_A \rangle \langle N_B \rangle$ and in effect that the covariance $\langle N_A N_B \rangle - \langle N_A \rangle \langle N_B \rangle \neq 0$

Therefore average levels of the two SQUID rings are correlated.

Model - Analysis

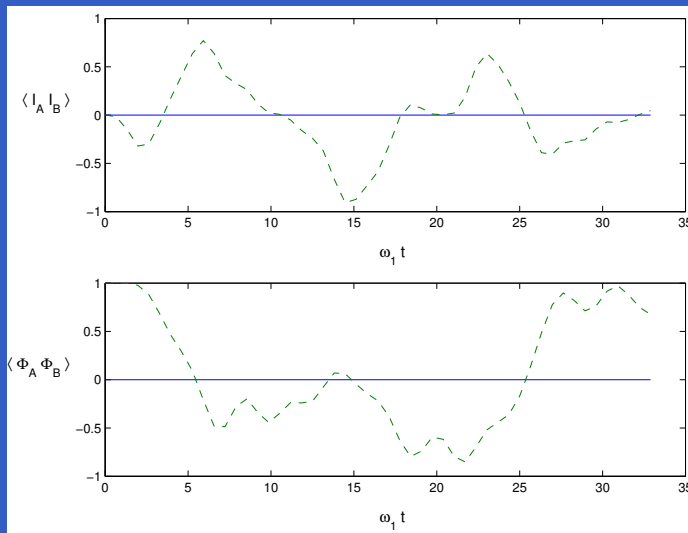
Bottom graph:



$\langle N_{F_1} N_{F_2} \rangle$ (solid line) and $\langle N_{F_1} \rangle \langle N_{F_2} \rangle$ (broken line) as functions of dimensionless time. It is seen that $\langle N_{F_1} N_{F_2} \rangle$ is different from $\langle N_{F_1} \rangle \langle N_{F_2} \rangle$

This is expected because the electromagnetic field is produced to be correlated.

Results



Top graph: Josephson currents in two SQUID rings as functions of dimensionless time $\omega_1 t$

Bottom graph: the electromagnetically induced magnetic fluxes as functions of dimensionless time $\omega_1 t$.

At $t = 0$ the system is in the state of Eq. (2) where ρ_F is in the separable state of Eq. (11) (solid line); and the entangled state of Eq. (13) (broken line).

The results show that in the case of entangled electromagnetic fields the Josephson currents are also entangled.



Entropic Criterion

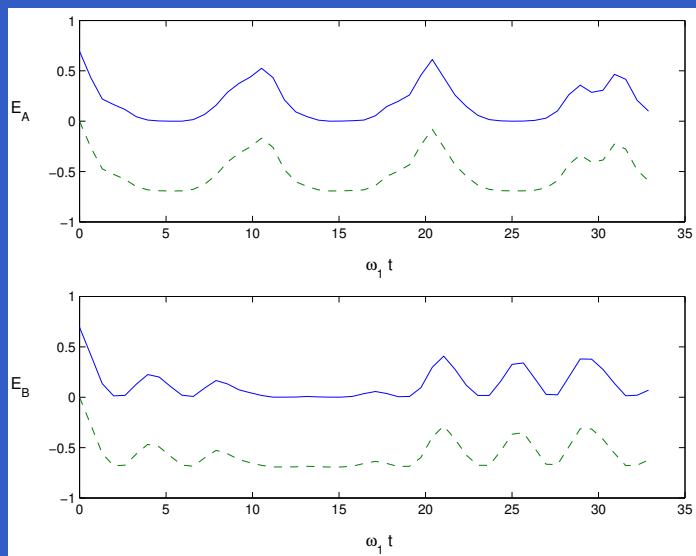
Calculated the conditional entropies

$$E_A = S(\rho_S) - S(\rho_A); \quad E_B = S(\rho_S) - S(\rho_B) \quad (15)$$

where $S(\rho) = -\text{Tr}(\rho \ln \rho)$ as criterion for entanglement. When $E_i < 0 (i = A, B)$ the system is entangled (although the converse is not true, i.e. an entangled system might have $E_i > 0$).



Results



E_A (Top) and E_B (Bottom) as functions of dimensionless time $\omega_1 t$. The system at $t = 0$ is in the state of Eq. (2) where ρ_F is in the separable state of equation Eq. (11) (solid line) and in the entangled state of equation Eq. (13) (broken line).

In the case of entangled electromagnetic fields these quantities are negative and therefore the SQUID rings are entangled.



Conclusions

- Started from two distant SQUID rings and a two-mode electromagnetic field,
- Considered two cases for the microwaves (i) initial state separable (ii) entangled,
- Studied the evolution of the system when one mode of the field interacts with the first SQUID and the second mode with the other SQUID.



Conclusions

- Shown how the entanglement of two-mode microwaves causes entanglement between the Josephson currents in two *distant* SQUIDs to become correlated.
- This line of work is useful in the general context of quantum communications.



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Questions

