Monte Carlo method for a superconducting Cooper-pair-box charge qubit measured by a single-electron transistor

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A Monte Carlo method which allows one to follow each electron tunneling event has been successfully applied to simulate transport properties of a single-electron transistor (SET) or more complicated single electronics circuits. This method gives physical insight into the processes taking place in the simulated system, and macroscopic ensemble properties can be calculated using large ensembles of single electron tunneling events. But to our knowledge, it has not yet been formally applied to quantum measurement problems by a SET detector. Here, we provide such an investigation and derive the *quantum-jump* [1,2] stochastic master equation (or quantum trajectory equation) for a single superconducting Cooper-pair-box (SCB) charge qubit (generalization to other charge qubit case is simple) continuously measured by a SET [3]. We also show that the master equation for the “partially” reduced density matrix presented in Ref. [3] (referred to here as a “partial” course-grain description) can be obtained by taking a “partial” average on the stochastic master equation over the fine grained measurement records of the tunneling events in the SET.

If a measurement is made on the qubit system and the results are available, the state or density matrix is conditioned on the measurement results. If the subsequent system evolution after the measurement is concerned, the conditional, stochastic master equations derived here should be employed. A set of typical quantum trajectories (stochastic state evolutions) and its corresponding measurement record are shown in Fig. 1 to illustrate the conditional dynamics. We consider the case that the leading tunneling process in the SET are sequential transitions between two adjacent charge states N=0 and N+1=1 on the SET island. $dN_{L/R,c}$ represents the number (either zero or one) of electron tunneling events seen in the infinitesimal time dt through the left and right junctions of the SET, respectively. The subscript c indicates that the quantity to which it is attached is conditioned on previous measurement results. As expected, due to Coulomb blockade, the SET measurement record of the randomly distributed moments of detections shown in Fig. 1(i) and (j) is in the order of an exactly alternating $dN_{Lc}=1$ and $dN_{Rc}=1$ time sequence. The joint reduced density matrix operator of the SCB qubit and extra charge on the SET island can be written as $R_c=\rho_{N,c}\,|N\rangle\langle N|+\rho_{N+1,c}\,|N+1\rangle\langle N+1|$, where $\rho_{N/N+1,c}(t)$ is each a 2x2 operator in the qubit basis. The conditional qubit density matrix operator alone can be obtained by writing $\rho_c(t)=\mathrm{Tr}_N[R_c(t)]=\rho_{N,c}(t)+\rho_{N+1,c}(t)$. Figure 1(a)-(d) show the time evolution of the joint SCB-qubit and SET-island diagonal density matrix elements conditioned on the measurement results of Fig. 1(i) and (f). The conditional evolutions of the qubit alone shown in Fig. 1(e) and (f) can be obtained from the sum of the joint state evolutions of (a) and (b), or (c) and (d), respectively. The probabilities, $P_{0/1,c}=\mathrm{Tr}_{\mathrm{qb}}[\rho_{0/1,c}]=\rho^{00}_{0/1,c}+\rho^{11}_{0/1,c}$ of the SET island state alone in (g) and (h) can be obtained by summing the evolutions in (a) and (c), or (b) and (d), respectively. Each quantum trajectory mimics a single history of the qubit state in a single run of the continuous measurement experiment. The conditional evolutions in (a)-(h) differ considerably from their ensemble average counterparts.

If only one measurement value is recorded in each run of experiments (e.g., the number of electrons N_R that have tunneled into the right lead or the drain in time *t*) and ensemble average properties (e.g, $P(N_R, t)$ the probability distribution of finding N_R in time *t*) are studied over many repeated experiments, the quantum trajectory approach presented here will give the same result as the master equation approach of the ``partially'' reduced density matrix [3]. This is demonstrated in Fig. 2. However, the possible individual realizations of quantum trajectories and their corresponding measurement records (e.g, in Fig. 1) do provide insight into, and aid in the interpretation of the average properties.

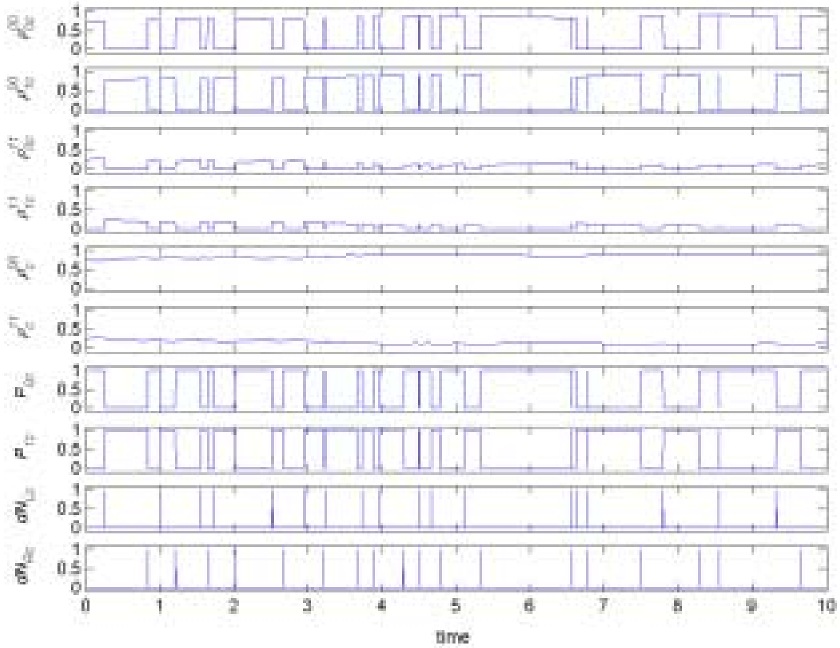

Figure 1. Conditional time evolution of the SCB charge qubit measured by the SET.

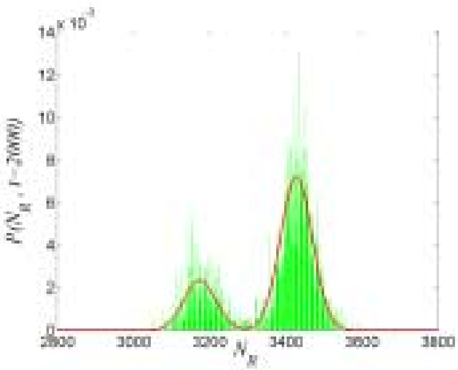

Figure 2. Probability distribution $P(N_R, t = 2000)$. Our simulation of the normalized histogram, using 2000 quantum trajectories and their corresponding detection records, is already in good agreement with the plot in solid line, obtained from the Fourier analysis of the partially reduced density matrix method described in Ref.[3].

[1] H.-S. Goan et al., *Phys. Rev. B* **63**, 125326 (2001); Phys. Rev. B **64**, 235307 (2001).
[2] H.-S. Goan, *Quant. Info. Comp.* **3**, 121 (2003).
[3] Y. Makhlin, A. Shnirman and G. Schon, *Rev. Mod. Phys.* **73**, 357 (2001).