Dynamical suppression of discrete noise in Josephson qubits

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Controlling the dynamics of a complex quantum system is at the heart of quantum information processing [1]. However in any real device the computational degrees of freedom entangle with the environment leading to decoherence [2]. Open-loop Bang-Bang (BB) quantum control techniques have been proposed as a way to achieve an effective decoupling from the environment [3, 4]. They may be operated by a sequence of very strong external pulses separated by a time Δt [3]. In the limit $\Delta t \to 0$ full decoupling [3, 4] of the unwanted interactions is achieved since the effective dynamics produced by them is averaged out by BB. The physics in this limit is a manifestation of the quantum Zeno effect [3, 5]. In practice Δt is finite especially if full-power pulses are generated. Thus decoupling is not perfect but the Zeno limit physics is still a good description if $\Delta t \ll \gamma^{-1}$, the typical time scale of the environment [3, 4]. If γ is large one may argue that BB chops noise and frequencies $\omega < 1/\Delta t < \gamma$ are averaged out. This optimistic scenario could foresee important applications to solid state coherent devices, where low-frequency noise [6] is one of the major problems for quantum state processing [7–9] and has been exploited in a series of recent papers [10–12].

Here we study environments of dissipative quantum bistable fluctuators [7], describing charge noise due impurities close to a solid-state qubit [7, 8, 13] which are responsible of a distictive phenomenology recently observed [14]. We consider a qubit ($\mathcal{H}_Q = -\frac{\varepsilon}{2}\,\sigma_z - \frac{\Delta}{2}\,\sigma_x$) coupled to an impurity. The Hamiltonian has the general form [12]

$$\mathcal{H} \;=\; \mathcal{H}_Q \;-\; \frac{1}{2}\,\sigma_z\,\hat{E} \;+\; \mathcal{H}_E \;+\; \mathcal{V}(t) \tag{1}$$

where $\mathcal{H}_E$ describes an impurity level occupied by a localized electron, tunneling to a fermionic band whith switching rate γ. The charge in the impurity is coupled to the qubit, $\hat{E} = v\hat{n}_{imp}$, and operates as a stray random bias of amplitude v. Control is operated by $\mathcal{V}(t)$, representing a sequence of instantaneous π-pulses about the $\hat{x}$ axis of the Bloch sphere. The evolution operator of the Hamiltonian (1) is $[\sigma_x \mathcal{S}]^{2N}$, where $\mathcal{S} = \exp(-i\mathcal{H}\Delta t)$ with $\mathcal{V}(t) = 0$ is the evolution between pulses. In order to take into account memory effects of the environment we denote with $\rho(t)$ the reduced density matrix (RDM) of the *qubit plus localized level* and use a standard master equation to find the map $\mathcal{E}_{\Delta t}[\rho(t)] = \rho(t+\Delta t) = \mathrm{e}^{\mathcal{L}\Delta t}\rho(t)$. After $t = 2N\Delta t$ we get $\rho(t) \approx \left[\mathcal{P}\cdot \mathrm{e}^{\mathcal{L}\Delta t}\right]^{2N}\rho(0)$ where $\mathcal{P}$ is the superoperator of the pulses. The RDM of the qubit $\rho^Q = \mathrm{Tr}_d[\rho(t)]$ is obtained by tracing out the localized level *at the end* of the protocol.

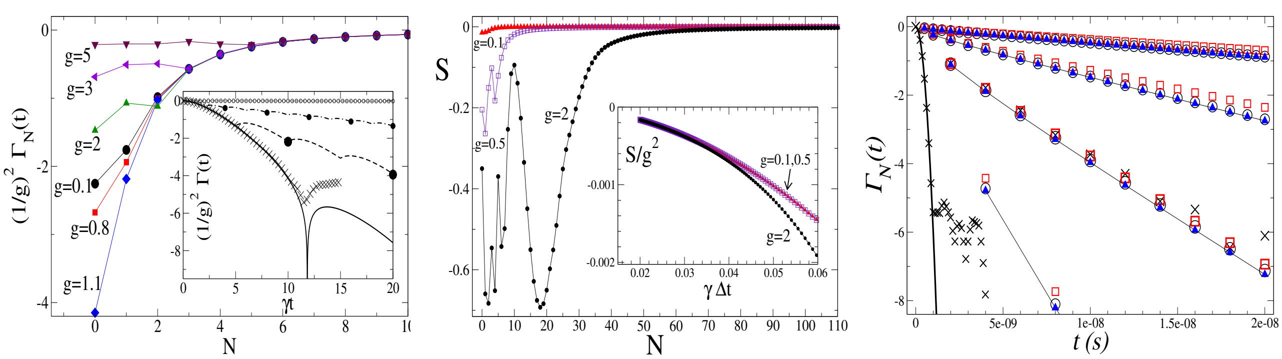

FIG. 1: **Left panel -** For fixed $t = 10\gamma^{-1}$ we plot $\Gamma_N(t)/g^2$ for BB procedures with N echoes. The parameter is $g \equiv v/\gamma$. A gaussian environment with the same power spectrum would give, for arbitrary g, the curve here labeled with $g = 0.1$. In the inset $\Gamma(t)$ for $g = 1.1$ without (thick line) and with pulses compared with results of the stochastic Schrödinger equation (crosses).
Central panel - We plot $S = \ln[\mathrm{Tr}(\rho^Q)^2]$ at fixed $t = 8\gamma^{-1}$ for protocols with N echo pair of pulses. Curves are parametrized by $g = (\Omega_1 - \Omega_0)/\gamma$. Here $\varepsilon = \Delta$, $v/\Omega = 0.2$. In the inset it is shown that S scales as g^2 for $N \gg 1$.
Right panel - BB control of $1/f$ noise for $\Delta = 0$. The analytic $\Gamma_N(t)$ at $t = 2N\Delta t$ (symbols) is compared with the evolution with no pulses (thick solid line). Noise is generated with N_{fl} fluctuators with rates γ_i distributed from $10^4\,Hz$ to $10^{10}\,Hz$. Noise level $\propto v^2 N_{fl}$ is *fixed* at a value typical of experiments in charge qubits: it is realized with coupling $v = 9.23\cdot 10^q\,Hz$, for $q = 6$ (full triangles), 7 (circles) and 8 (squares), with $N_{fl} = 6\cdot 10^{17-2q}$. Crosses are stochastic Schrödinger simulations with 10^5 realizations of the $1/f$ process, for $q = 7$.

The major step of this program is the diagonalization of the 8×8 supermatrix $\left[\mathcal{P} \cdot e^{\mathcal{L}\Delta t}\right]^2$.

In some cases the calculation can be carried out analytically, as for $\Delta = 0$ [12]. We find that decoupling of this environment is sensitive to details of its dynamics. If pulses are not very frequent it shows a rich variety of behaviors, suggesting that BB may also be used for spectroscopy of a solid-state environment. An important parameter is $g = (\Omega_1 - \Omega_0)/\gamma$, where Ω_i are the splittings of the qubit when the fluctuator sits in state i, which quantifies the non-gaussianity of the environment [7]. Slow fluctuators ($g > 1$) are responsible for inhomogeneuos broadening and for a physics dominated by memory effects where decoherence depends strongly on details of the protocols. We study the decay of the coherences of the qubit, given by $\Gamma_N(t) = \ln\left|\frac{\rho_{ab}(t)+\rho_{cd}(t)}{\rho_{ab}(0)+\rho_{cd}(0)}\right|$ (Fig. 1 left). At any fixed $\bar{t} = 2N\Delta t$, $|\Gamma_N(\bar{t})|$ decreases monotonically when the pulse frequency $1/\Delta t$ increases, which shows that BB effectively suppresses RTN and in this regime $|\Gamma_N(\bar{t})| \sim g^2$ shows universal behavior. On other hand for $2N \ll \gamma t$ qualitative differences in the behavior are apparent for $g < 1$ and $g > 1$. The physics for $\Delta \neq 0$ is even richer. We study the degree of purity $S = \ln \mathrm{Tr}(\rho^Q)^2$, efficient decoupling corresponding to $S = 0$, i.e. localization in a "Zeno subspace" [5] which is a pure state. Results (Fig. 1 central) show that for frequent pulses $S \approx 0$, but decoupling of a slow fluctuator, $g > 1$, requires a very large N. Again we find universal behavior, $S \sim g^2$ (inset of Fig. 1 central). Instead for a smaller number of pulses it may happen, especially for $g > 1$ that S is not monotonic with N, including the possibility that the qubit decays *faster* than in absence of pulses [3], reminiscent of the *inverse* Zeno effect [5].

In order to treat $1/f$ noise we now extend our formalism to a "multi-mode" environment. We first generalize the results at $\Delta = 0$ of Ref. [3], to an arbitrary (non-gaussian) environment. For the evolution between two pulses at t_1 and t_2 we can use $[\mathcal{H}, \sigma_z] = 0$ and following the same steps of Ref. [3] we can write the evolution operator for a BB procedure at $t = 2N\Delta t$ as $\mathcal{S}_{2N}(t) = \left[e^{-i(\mathcal{H}_E + \frac{1}{2}\sigma_z \hat{E})\Delta t} e^{-i(\mathcal{H}_E - \frac{1}{2}\sigma_z \hat{E})\Delta t}\right]^N$. In the overall BB procedure σ_z is conserved, so we need only off diagonal entries of the RDM of the qubit, in the σ_z basis $\rho^Q_{\sigma\sigma'}(t) = \rho^Q_{\sigma\sigma'}(0)\, \mathrm{Tr}_E\left\{\mathcal{S}_{2N}(t|\sigma)\, w_E\, \mathcal{S}^\dagger_{2N}(t|\sigma')\right\}$ where we assumed factorized initial conditions. Here $\mathcal{S}_{2N}(t|\sigma) = \langle\sigma|\mathcal{S}_{2N}(t)|\sigma\rangle$ expresses the conditional evolution of the environment under a well defined sequence of $\sigma = \pm 1$. The trace factorizes if the environment is made of noninteracting "modes", so we can apply this result to a set of noninteracting impurities originating $1/f$ noise. In particular the decay of the coherences is $\Gamma_N(t) = \sum_\eta \ln\left|\frac{\rho^{(\eta)}_{ab}(t)+\rho^{(\eta)}_{cd}(t)}{\rho_{ab}(0)+\rho_{cd}(0)}\right|$. Results in the right panel of Fig. 1 show that frequent pulses (curves with many symbols) drastically reduce decoherence. For noise levels typical of experiments with charge qubits (Fig. 1 right) the pulse rate for substantial recovery is practically independent on v. The situation may change if a broad distribution of the couplings v is considered [10]. The physics is richer for $\Delta \neq 0$ and possibly compensation of $1/f$ noise is non monotonic for increasing pulse frequency, as it happens for a single impurity.

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