Size dependence of the superconductor-insulator transition in Josephson junction arrays

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The superconductor-insulator (SI) transition in a single Josephson junction (JJ) and JJ arrays in one and two dimensional geometry have been extensively studied. Experimentally observed phase diagrams in the E_J/E_C - g plane show two features [1-5]. Here E_J, E_C and $g=R_Q/R_S$ are, respectively, the Josephson energy, capacitive Coulomb energy of the superconducting islands, and the Ohmic shunt resistance in units of the quantum resistance, $R_Q=h/4e^2$. One is dissipative transition controlled by g and the other is quantum phase transition controlled by E_J/E_C. Here our focus is on the E_J/E_C axis at g=0 where the critical E_J/E_C~3 for a single JJ and ~0.3 for the 48x40 JJ arrays. This order of magnitude difference was argued to be an evidence of cooperative phenomena and the phase boundary in the 48x40 JJ arrays a true phase boundary.

The theoretically irrefutable treatment of the fully 2D quantum mechanical and strongly correlated systems near the phase boundary is an ever-lasing challenge. One simplification is to neglect the quantum fluctuation due to the capacitive Coulomb energy and assume that the single parameter in the resulting 2D XY model E_J/T where T is temperature may play the role of E_J/E_C at T=0. The difference between the fully quantum mechanical JJ and the classical 2D XY model is that the former undergoes a quantum phase transition (QPT) whereas the latter a classical phase transition (CPT). The qualitative resemblance nevertheless lies in the fact that both undergo the Berezinskii-Kosterlitz-Thouless (BKT) transition. More importantly, the strong size dependence seems to be rather universal in two dimension. In fact, while the 2D XY model has been studied extensively in the past, there are two noteworthy development recently concerning the strong size dependence. One is in the context of the planar magnets. It is well-known that in spite of the fact that the BKT transition describing the SI transition is infinite order, in a dozen of planar magnets, the magnetization appears like in the 2-nd order transition with a universal magnetization exponent ~0.23. Bramwell and Holdsworth [6] used renormalization group, Monte Carlo and finite-size scaling to show that the transition looks like 2-nd order with Tc~1.08 for 10^3 spins and ~1.02 for 10^4 spins, which are fairly modified from Tc=0.892 believed to be exact for the infinite system. The size dependence is significant, reflecting the peculiar singularity of the correlation length in the BKT transition. The other development is the transfer-matrix density-matrix-renormalization-group (TM-DMRG) method by the present author [7]. Using the Roomany-Wyld finite-size scaling, without any unknown parameters, nor relying on the BKT-RG, we found that Tc=1.07 for 2000-3000 spins in agreement with [6].

To account for the strong size dependence in the critical $E_J/E_C \sim E_J/T$, we need to go one step further and look for a signature of the SI transition as a function of the system size. Note that the 2D XY model is, under Villain approximation, equivalent to the 1D quantum sine-Gordon (SG) model, with the correspondence $E_J/E_C \sim 1/\beta^2$ [8],

$$H = \sum_{i=1}^{L} \{ -\frac{\beta^2}{2}\frac{d^2}{d\phi_i^2} + \frac{1}{2\beta^2}(\phi_i - \phi_{i+1})^2 + \frac{1}{\beta^2}(1+\cos\phi_i) \}$$

The ground state of the SG Hamiltonian can be analyzed by DMRG. We have found that there is a spontaneous symmetry breaking (SSB) of the ground state as a function of the system size [9]. The identification of the BKT transition as a SSB is then demonstrated by combining DMRG with the Roomany-Wyld finite-size scaling. Fig.1 is the ground state phase diagram in the β^2-L plane. With increase of the size, $E_J/E_C \propto 1/\beta^2$ decreases like $\beta^2(L=5)/\beta^2(L=50)$ ~0.1 which is consistent with the experimental findings.

After this is done, it is now clear that we can accurately analyze the size dependences of (1) the QPT in the JJ chain with on-site Coulomb fluctuation and (2) the CPT in the 2D XY model for the JJ arrays with relatively large superconducting islands. We will report on the size dependence of the superfluid densities of these systems.

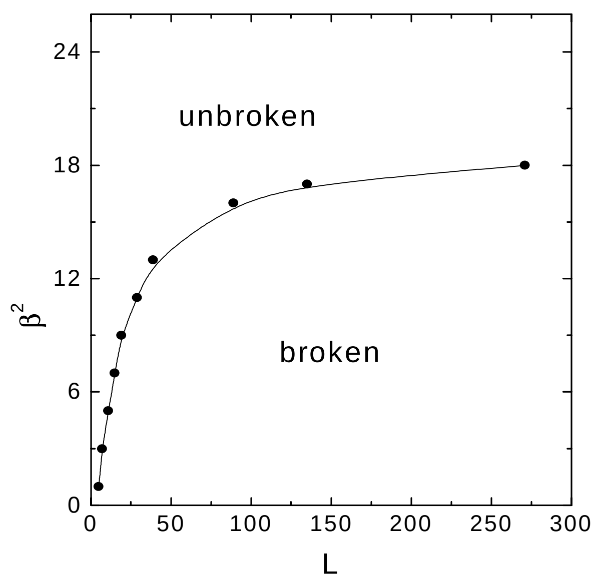

FIG.1 The ground-state phase diagram in the β^2-L plane

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